Dynamical vector-valued optimization of complex adaptive control of retail unit of a commercial bank

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Dynamical Vector-Valued Optimization of Complex Adaptive Control of Retail Unit of a Commercial Bank

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Abstract. In this article discusses current issues of optimizing the management of the retail unit of a commercial bank in order to improve the economic efficiency and competitiveness of the bank. The article presents the method of solving a multi-step dynamical problem of complex adaptive control of the number of employees and sales of the Bank's Retail Unit with a vector quality criterion for the control process under consideration. For the organization of optimal adaptive terminal control of the system under study, a recurrent algorithm is proposed, which reduces the initial multi-step problem to the realization of a finite sequence of tasks for an optimal program terminal control. On the basis of the developed computer simulation system and the results of computer simulation, the choice of the optimal solution for the optimization problem under consideration was made. The proposed approach allows us to develop management solutions based on the availability of current information support, that is, using the feedback principle, aimed at creating the optimal number of employees and the sales system of the Bank's Retail Unit, which helps optimize profits and gain competitive advantages for the bank.

INTRODUCTION

Nowadays the issue of managing the effectiveness of banking is quite topical. In the conditions of an unstable financial situation, and also taking into account the fulfillment of the necessary requirements of the supervisory authorities, the management of any credit institution is trying to minimize risks and increase the profitability of its business. In addition, the banking business is highly dynamic, digital technologies are being actively introduced, and remote sales channels are being developed. Despite the reduction in the number of credit institutions in Russia, competition is still quite high. Banks are introducing new products and services, changing forms of service, and increasing customer requirements. Accordingly, existing banks are constantly forced to improve their operations in order to retain existing customers and acquire new ones. As a result of these changes and the influence of other factors, banks change their approaches and business strategies, and they must make such decisions and act very quickly in order to survive in the competition with other banks. Competition in the banking environment is the rivalry of commercial and non-bank credit organizations in order to ensure a leading position in the banking services market. Increasing banking competition and increasing customer requirements for banking services stimulate the improvement of marketing methods and banking management [1], [2].

One of the key areas on the path to improving business efficiency is resource management, which includes the process of personnel management. Due to the rapid development of digital sales channels, there is an acute question about the profitability and feasibility of the existence and opening of new offices of the bank, expansion or reduction of staff working with customers. In most cases, these activities are carried out on the basis of expert opinion of decision makers in a credit institution. Sometimes these decisions are based on the results of one-time calculations of the economic effect of the events held. In this regard, the presence in the bank of a developed information and analytical decision support system, which allows you to determine as fully and as quickly as possible all the consequences of

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the possible options for the solutions under consideration for choosing the optimal one, has a significant impact on the results of implementing these decisions.

The management support system for financial management decisions is a set of tasks, methods (techniques), software and technical systems interconnected by goals, parameters and conditions, which allow to form a set of reporting forms containing information for both management decisions and options such decisions [3]. The most effective use of these systems is facilitated by the use of economic-mathematical models and optimal control methods for the formation of the best possible financial decisions and support for making effective management decisions.

CONSTRUCTION OF THE ECONOMIC-MATHEMATICAL MODEL OF MANAGING A RETAIL UNIT OF A COMMERCIAL BANK

Let us describe the procedure for the formation of a dynamic economic-mathematical model within the framework of research into the processes of optimizing the management of the bank's retail business. Similar mathematical models for economic systems are presented, for example, in [4], [5]

Consider the input data and features of the management decision-making process for use in the retail division of the bank in the event of a change in the number of employees and setting of sales standards for their various roles. Similar models for banking, only in a more simplified form, were considered in [6], [7] .

To form an economic-mathematical model of the decision-making process in banking, we introduce the following notation:

n is the number of basic banking "portfolio" products for individuals (for example, mortgage lending, car loans, deposits to individuals, debit bank cards, etc.; $n \in \mathbb{N}$; where $\mathbb{N}$ is here and below, the set of all natural numbers);

m is the total number of types of positions of employees implementing these products (the functions of some roles contain responsibilities for the sale of several types of products, others for only one product; $m \in \mathbb{N}$);

$x(t) = (x_1(t), x_2(t), \dots, x_n(t))' \in \mathbb{R}^n$ is the vector characterizing the volume of the portfolio of each of the banking products in the period of time t in thousands of rubles ($t \in \overline{0, T-1} = \{0, 1, 2, \dots, T-1\}$; $T \in \mathbb{N}$) , for which each i-th coordinate $x_i(t)$ is the value of the portfolio volume of the i-th type of banking product ($i \in \overline{1, n}$); here and below for $k \in \mathbb{R}^k$, $\mathbb{R}^k$ is the k-dimensional Euclidean vector space of column vectors; T is the number of months defining the time interval $\overline{0, T}$ over which the process under consideration is controlled.

$y(t) = (y_1(t), y_2(t), \dots, y_m(t))' \in \mathbb{R}^m$ is the vector characterizing the number of different categories of employees in the bank (number of people) in a period of time t ($t \in \overline{0, T-1}$), where each j-th coordinate $y_j(t)$ is the value of the staff number of the j-th type of position ($j \in \overline{1, m}$);

$A(t) = \|a_{ij}(t)\|$, $i \in \overline{1, n}$, $j \in \overline{1, m}$ is the matrix of sales standards per month in the period of time t ($t \in \overline{0, T-1}$), $a_{ij}(t)$ is the normative number of products sold i-th type of employee j-th category ($i \in \overline{1, n}$, $j \in \overline{1, m}$);

$H = (h_1, h_2, \dots, h_n)' \in \mathbb{R}^n$ is the vector of the monthly repayment coefficients for each of the product types in the portfolio, %;

$S = (s_1, s_2, \dots, s_n)' \in \mathbb{R}^n$ is the vector of average values of products sales, thousand rubles;

$u(t) = (u_1(t), u_2(t), \dots, u_m(t))' \in \mathbb{R}^m$ is the vector of input of the number of each category of employees (number of people), in the period of time t ($t \in \overline{0, T-1}$), in which each j-th coordinate $u_j(t)$ is the value of the number of input staff members of the j-th type of position ($j \in \overline{1, m}$).

The dynamics of the number of employees of the Bank's Retail Unit and the portfolio of banking products for individuals, depending on the sales standards, is described by a system of quasilinear recurrent equations of the form:

$$\begin{cases} y_j(t+1) = y_j(\tau_t) + u_j(\tau_t), u_j(0) = 0, \tau_t = \tau \cdot \boldsymbol{E}(\frac{t}{\tau}), y_j(\tau_0) = y_j(0) = y_{0j}, j \in \overline{1, m}, \\ x_i(t+1) = x_i(t) - h_i \frac{x_i(t)}{100} + s_i \sum_{j=1}^{m} a_{ij}(t)[y_j(\tau_t) + u_j(\tau_t)], x_i(0) = x_{0i}, i \in \overline{1, n}, \\ t \in \overline{0, T-1}, h_i = const, s_i = const, i \in \overline{1, n}, \end{cases} \tag{1}$$

where τ is the duration of the period of time unchanged number of the official staff of the Retail Unit of the Bank and sales standards ($\tau \in \mathbb{N} : \tau \leq T$);

$\boldsymbol{E} : \mathbb{R}^1 \to \mathbf{Z} = \mathbb{N} \cup \{0\}$ is the function of integer part of the real number.

The vector of input of the number of employees $u(\tau_t) = (u_1(\tau_t), u_2(\tau_t), \dots, u_m(\tau_t))' \in \mathbb{R}^m$ and the matrix of sales standards $A(\tau_t) = \|a_{ij}(\tau_t)\|, i \in \overline{1, n}, j \in \overline{1, m}$, in the framework of the Bank's Retail Unit in the period of time

t ($t \in \overline{0, T-1}$) are the control actions (controls) in the system of equations (1), for which the following specified constraints are necessary:

$$\begin{gathered}\overline{u}(t) = (u(\tau_t), A(\tau_t)) \in \overline{\boldsymbol{U}}_1(t, y(\tau_t), y(\tau_t - 1)) = (\boldsymbol{U}_1(\tau_t) \times \boldsymbol{A}_1(\tau_t)) \subset \mathbb{R}^{\overline{p}} \\ (\overline{p} = m(n+1) \in \mathbb{N}), \\ u(\tau_t) \in \boldsymbol{U}_1(\tau_t) \subset \mathbb{R}^m, t \in \overline{0, T-1}, \\ A(\tau_t) \in \boldsymbol{A}_1(\tau_t) \subset \mathbb{R}^{n \times m}.\end{gathered} \tag{2}$$

More detailed description of constraints (2) is presented in [8].

In the management process, for all $t \in \overline{0, T-1}$ the following predetermined phase constraints must also be satisfied:

$$\overline{x}(t) = (y(t), x(t)) \in \overline{\boldsymbol{X}}_1(t) = \boldsymbol{Y}_1(t) \times \boldsymbol{X}_1(t), \tag{3}$$

$$\begin{gathered} y(t) \in \boldsymbol{Y}_1(t), x(t) \in \boldsymbol{X}_1(t), \\ \boldsymbol{Y}_1(t) = \{y(t) : y(t) = (y_1(t), y_2(t), \ldots, y_m(t))' \in \mathbb{R}^m, \forall i \in \overline{1, m} : y_i(t) \geq 0\}, \\ \boldsymbol{X}_1(t) = \{x(t) : x(t) = (x_1(t), x_2(t), \ldots, x_n(t))' \in \mathbb{R}^n, \forall i \in \overline{1, n} : x_i(t)) \geq 0\}, \end{gathered} \tag{4}$$

where $\overline{x}(t) = (y(t), x(t))' \in \mathbb{R}^{\overline{n}}$ is the phase vector of a discrete dynamic system (1); $\overline{n} = m + n$.

Within the framework of the problem to be solved, we will establish a constraint that the control action $\overline{u}(t) = (u(\tau_t), A(\tau_t)) \in \overline{\boldsymbol{U}}_1(t)$, $t \in \overline{0, T-1}$, may change no more than once every 6 months, i.e. at $\tau \geq 6$.

For the considered integer time interval $\overline{0, T}$, a fixed period of time $\vartheta \in \overline{0, T-1}$ ($T \in \mathbb{N}$) and the corresponding integer time interval $\overline{\vartheta, T} \subseteq \overline{0, T}$, we denote $\overline{x}(T) = \varphi_{\overline{\vartheta, T}}(T; \overline{x}(\vartheta), \overline{u}(\cdot))$ the final state (at time T) of the trajectory $\overline{x}(\cdot) = \{\overline{x}(t)\}_{t \in \overline{\vartheta, T}} = \{y(t), x(t)\}_{t \in \overline{\vartheta, T}}$ of the phase vector of the discrete dynamic system (1) describing the dynamics of the considered optimization process on the time interval $\overline{0, T}$ corresponding to the set $(\overline{x}(\vartheta), \overline{u}(\cdot))$, where $\overline{x}(\vartheta) = (y(\vartheta), x(\vartheta)) \in \overline{X}_1(\vartheta) = Y_1(\vartheta) \times X_1(\vartheta)$ ($\overline{X}_1(0) = Y_1(0) \times X_1(0) = \{\overline{x}(0)\} = \{y(0)\} \times \{x(0)\} = \{y_0\} \times \{x_0\} = Y_{01} \times X_{01}$), $\overline{u}(\cdot) = \{\overline{u}(t)\}_{t \in \overline{\vartheta, T-1}}, \forall t \in \overline{\vartheta, T-1} : \overline{u}(t) \in \overline{\boldsymbol{U}}_1(t, y(\tau_t), y(\tau_t - 1))$.

We describe the information capabilities of the control subject - the manager P of the Retail Unit of the Bank in the process of adaptive (on the basis of feedback) control in a discrete dynamic system (1)-(4).

Let on the considered integer time interval $\overline{0, T}$ for any $\vartheta \in \overline{0, T-1}$ ($T \in \mathbb{N}$) and the corresponding integer time interval $\overline{\vartheta, T} \subset \overline{0, T}$, to the time point ϑ in the process of adaptive control by the manager P the following values are measured and stored: $\overline{x}(\vartheta) = \overline{x}_\vartheta$ - the phase state of the control object during the control period ϑ ($\overline{x}(0) = \overline{x}_0$); $\overline{u}(\cdot) = \{\overline{u}(t)\}_{t \in \overline{0, \vartheta-1}}$- the history of the implementation of the admissible control of the manager P on the time interval $\overline{0, \vartheta}$. It is also assumed that the system of equations of dynamics (1) of the control object and constraints (2)–(4) for it are also known.

The quality functional for the considered process of adaptive control is the vector functional $\boldsymbol{F}_{\overline{\vartheta, T}} = (\boldsymbol{F}^{(1)}_{\overline{\vartheta, T}}, \boldsymbol{F}^{(2)}_{\overline{\vartheta, T}}, \boldsymbol{F}^{(3)}_{\overline{\vartheta, T}}) : \mathbb{R}^{\overline{n}} \times \mathbb{R}^{\overline{n} \times (T - \vartheta + 1)} \times \mathbb{R}^{\overline{n} \times (T - \vartheta + 1)} \to \mathbb{R}^1$, where the first functional $\boldsymbol{F}^{(1)}_{\overline{\vartheta, T}} : \mathbb{R}^{\overline{n}} \to \mathbb{R}^1$ evaluates the size of the profit of the Retail Bank's Unit received over a period of time $\overline{\vartheta, T}$, the second functional $\boldsymbol{F}^{(2)}_{\overline{\vartheta, T}} : \mathbb{R}^{\overline{n} \times (T - \vartheta + 1)} \to \mathbb{R}^1$ - Cost Income Ratio (CIR) - evaluates the ratio of operating costs to the operating income of the Retail Bank's Unit over a period of time $\overline{\vartheta, T}$, the third functional $\boldsymbol{F}^{(3)}_{\overline{\vartheta, T}} : \mathbb{R}^{\overline{n} \times (T - \vartheta + 1)} \to \mathbb{R}^1$ evaluates the increase in the share in the market of attracted funds from individuals over a period of time $\overline{\vartheta, T}$. Detailed description of the functionals and the order of their calculation are presented in [8]. Moreover, the values of the vector functional $\boldsymbol{F}_{\overline{\vartheta, T}}$ are determined on admissible (possible) realizations of the phase trajectory $\overline{x}(\cdot) = (y(\cdot), x(\cdot))' = \varphi_{\overline{\vartheta, T}}(\cdot; \overline{x}(\vartheta), \overline{u}(\cdot))$ of the system (1)–(4) on the time interval $\overline{\vartheta, T}$, which correspond to the admissible realizations of the sets $(\overline{x}(\vartheta), \overline{u}(\cdot))$, where $\overline{x}(\vartheta) \in \overline{\boldsymbol{X}}_1(\vartheta) = \boldsymbol{Y}_1(\vartheta) \times \boldsymbol{X}_1(\vartheta)$, $\overline{u}(\cdot) = \{\overline{u}(t)\}_{t \in \overline{\vartheta, T-1}}, \forall t \in \overline{\vartheta, T-1} : \overline{u}(t) \in \overline{\boldsymbol{U}}_1(t, y(\tau_t), y(\tau_t - 1))$.

Then, within the framework of the formed discrete dynamic system (1)–(4), the goal of optimal adaptive control from the point of view of the manager P of the Bank's Retail Unit can be formulated as follows: at a given time interval $\overline{0, T}$, it is required that manager P organizes his control $\overline{u}(\cdot) = \{\overline{u}(t)\}_{t \in \overline{0, T-1}}$ ($\forall t \in \overline{0, T-1} : \overline{u}(t) \in \overline{\boldsymbol{U}}_1(t, y(\tau_t), y(\tau_t - 1))$) according to the feedback principle (as the implementation of the optimal adaptive strategy from the selected class of admissible adaptive strategies), using the implementation of the values of the phase vector $\overline{x}(\cdot) = \{\overline{x}(t)\}_{t \in \overline{0, T}}$ possible by (1)–(4), together with all other access for him information about the process, so that value of the vector functional $\boldsymbol{F}_{\overline{0, T}} = \boldsymbol{F}_{\overline{0, T}}(\overline{x}(\cdot))$, defined on the implementation of the trajectory $\overline{x}(\cdot) \in \mathbb{R}^{\overline{n} \times (T - \vartheta + 1)}$, would be the maximum (where

$\overline{x}(\cdot) = (y(\cdot), x(\cdot))' = \varphi_{\overline{\vartheta,T}}(\cdot; \overline{x}(\vartheta), \overline{u}(\cdot))$ is a realization of the trajectory of the control object described system (1) on the time interval $\overline{\vartheta, T}$ corresponding implementation allowable set $(\overline{x}(\vartheta), \overline{u}(\cdot))$).

FORMALIZATION OF OPTIMAL PROGRAM AND ADAPTIVE CONTROL PROBLEMS

We call a set $w(\vartheta) = \{\vartheta, \overline{x}(\vartheta)\} \in \overline{0,T} \times \mathbb{R}^{\overline{n}}$ ($w(0) = w_0 = \{0, \overline{x}_0\}$) ϑ- position of manager P in a discrete dynamic system (1)–(4). For each $\vartheta \in \overline{0,T}$ we also define the set $\hat{\boldsymbol{W}}(\vartheta) = \{\vartheta\} \times \mathbb{R}^{\overline{n}}$ ($\hat{\boldsymbol{W}}(0) = \hat{\boldsymbol{W}}_0 = \{w(0) = w_0 : w_0 = \{0, \overline{x}_0\} \in \{0\} \times \mathbb{R}^{\overline{n}}\}$) of all admissible ϑ-positions of manager P. On the considered integer time interval $\overline{0,T}$ for a fixed $\vartheta \in \overline{0, T-1}$ ($T \in \mathbb{N}$) and corresponding integer time interval $\overline{\vartheta, T} \subseteq \overline{0,T}$, on the basis of (2)–(4) for each admissible ϑ-position $w(\vartheta) = \{\vartheta, \overline{x}(\vartheta)\} \in \hat{\boldsymbol{W}}(\vartheta)$ of manager P, we determine the final set $\overline{\boldsymbol{U}}(\overline{\vartheta,T}, w(\vartheta)) = \{\overline{u}(\cdot)\} = \{(u(\cdot), A(\cdot))\}$ of admissible program controls $\overline{u}(\cdot) = (u(\cdot), A(\cdot)) = (\{u(\tau_t)\}_{t \in \overline{\vartheta, T-1}}, \{A(\tau_t)\}_{t \in \overline{\vartheta, T-1}})$ of manager P corresponding to the time interval $\overline{\vartheta, T}$ by the following relation:

$$\begin{gathered}\overline{\boldsymbol{U}}(\overline{\vartheta,T}, w(\vartheta)) = \{\overline{u}(\cdot) : \overline{u}(\cdot) = (u(\cdot), A(\cdot)) = (\{u(\tau_t)\}_{t \in \overline{\vartheta, T-1}}, \{A(\tau_t)\}_{t \in \overline{\vartheta, T-1}}) \subset \mathbb{R}^{\overline{p} \times (T-\vartheta)}, \\ \forall t \in \overline{\vartheta, T-1}, (u(\tau_t), A(\tau_t)) \in \overline{\boldsymbol{U}}_1(t, y(\tau_t), y(\tau_t - 1)) = (\boldsymbol{U}_1(\tau_t) \times \boldsymbol{A}_1(\tau_t))\}.\end{gathered} \tag{5}$$

Then for each fixed integer time interval $\overline{\vartheta, T}$ and valid implementation options for the sets $(w(\vartheta), \overline{u}(\cdot)) \in \hat{\boldsymbol{W}}(\vartheta) \times \overline{\boldsymbol{U}}(\overline{\vartheta,T}, w(\vartheta))$, where $w(\vartheta) = \{\vartheta, \overline{x}(\vartheta)\} \in \hat{\boldsymbol{W}}(\vartheta)$ ($w(0) = w_0$) are the ϑ-positions of manager P, and $\overline{u}(\cdot) = (u(\cdot), A(\cdot)) = (\{u(\tau_t)\}_{t \in \overline{\vartheta, T-1}}, \{A(\tau_t)\}_{t \in \overline{\vartheta, T-1}}) \in \overline{\boldsymbol{U}}(\overline{\vartheta,T}, w(t))$ is the program control of manager P allowed for this period of time, as indicators of quality (target functions) of adaptive control in a discrete dynamic system (1)–(4), describing the dynamics of the optimization process of the complex program control of the number of employees and the sales system of the Bank's Retail Unit, as well as the existing limitations, we introduce for consideration a generalized objective function $\boldsymbol{\Phi}_{\overline{\vartheta,T}}(w(\vartheta), \overline{u}(\cdot))$. The values of it for all possible implementations of the sets $(\overline{x}(\vartheta), \overline{u}(\cdot)) \in \overline{\boldsymbol{X}}_1(\vartheta) \times \overline{\boldsymbol{U}}(\overline{\vartheta,T}, w(\vartheta))$ the time interval $\overline{\vartheta, T}$ are determined in accordance with the ratio:

$$\begin{gathered}\boldsymbol{\Phi}_{\overline{\vartheta,T}}(w(\vartheta), \overline{u}(\cdot)) = \lambda_1 \cdot \boldsymbol{\Phi}^{(1)}_{\overline{\vartheta,T}}(w(\vartheta), \overline{u}(\cdot)) - \lambda_2 \cdot \boldsymbol{\Phi}^{(2)}_{\overline{\vartheta,T}}(w(\vartheta), \overline{u}(\cdot)) + \lambda_3 \cdot \boldsymbol{\Phi}^{(3)}_{\overline{\vartheta,T}}(w(\vartheta), \overline{u}(\cdot)) = \\ = \lambda_1 \cdot \boldsymbol{F}^{(1)}_{\overline{\vartheta,T}}(\varphi_{\overline{\vartheta,T}}(T; \overline{x}(\vartheta), \overline{u}(\cdot))) - \lambda_2 \cdot \boldsymbol{F}^{(2)}_{\overline{\vartheta,T}}(\varphi_{\overline{\vartheta,T}}(T; \overline{x}(\vartheta), \overline{u}(\cdot))) + \lambda_3 \cdot \boldsymbol{F}^{(3)}_{\overline{\vartheta,T}}(\varphi_{\overline{\vartheta,T}}(T; \overline{x}(\vartheta), \overline{u}(\cdot))) = \\ = \lambda_1 \cdot \boldsymbol{F}^{(1)}_{\overline{\vartheta,T}}(\overline{x}(T)) - \lambda_2 \cdot \boldsymbol{F}^{(2)}_{\overline{\vartheta,T}}(\overline{x}(T)) + \lambda_3 \cdot \boldsymbol{F}^{(3)}_{\overline{\vartheta,T}}(\overline{x}(T)) = \\ = \lambda_1 \cdot \boldsymbol{F}^{(1)}_{\overline{\vartheta,T}}(y(T), x(T)) - \lambda_2 \cdot \boldsymbol{F}^{(2)}_{\overline{\vartheta,T}}(y(T), x(T)) + \lambda_3 \cdot \boldsymbol{F}^{(3)}_{\overline{\vartheta,T}}(y(T), x(T)) = \boldsymbol{F}(\overline{x}(T)) = \boldsymbol{F}(y(T), x(T)), \\ \forall i \in \overline{1,3} : \lambda_i \geq 0; \textstyle\sum_{i=1}^{3} \lambda_i = 1,\end{gathered} \tag{6}$$

where $\boldsymbol{\Phi}^{(1)}_{\overline{\vartheta,T}}(w(\vartheta), \overline{u}(\cdot))$, $\boldsymbol{\Phi}^{(2)}_{\overline{\vartheta,T}}(w(\vartheta), \overline{u}(\cdot))$ and $\boldsymbol{\Phi}^{(3)}_{\overline{\vartheta,T}}(w(\vartheta), \overline{u}(\cdot))$ are the objective functions (functionals) described in papers [8]; λ_i, $i \in \overline{1,3}$ are the weighting factors which reflect the significance of each of the quality criteria in terms of influence on the result of the choice of the management strategy of the retail division of the bank.

The generalized objective function $\boldsymbol{\Phi}_{\overline{\vartheta,T}}(w(\vartheta), \overline{u}(\cdot))$ is a scalar convolution of a vector objective function (vector functional) $\boldsymbol{\Phi}_{\overline{\vartheta,T}}(w(\vartheta), \overline{u}(\cdot)) = (\boldsymbol{\Phi}^{(1)}_{\overline{\vartheta,T}}(w(\vartheta), \overline{u}(\cdot)), \boldsymbol{\Phi}^{(2)}_{\overline{\vartheta,T}}(w(\vartheta), \overline{u}(\cdot)), \boldsymbol{\Phi}^{(3)}_{\overline{\vartheta,T}}(w(\vartheta), \overline{u}(\cdot)))$. The method of scalarization of vector objective functions is described in detail in [8].

We assume that the manager P on the time interval $\overline{\vartheta, T} \subseteq \overline{0,T} (\vartheta < T)$ for each permissible implementation of his ϑ-position $w(\vartheta) = \{\vartheta, \overline{x}(\vartheta)\} \in \hat{\boldsymbol{W}}(\vartheta)$ is interested in such an outcome of the management process - by influencing him by the possible choice of his admissible program controls $\overline{u}(\cdot) \in \overline{\boldsymbol{U}}(\overline{\vartheta,T}, w(\vartheta))$ at which the functional $\boldsymbol{\Phi}_{\overline{\vartheta,T}}$ defined by relation (6) takes the greatest possible value.

The achievement of this goal of the manager P is realized in the framework of solving the following non-linear multi-step optimal program control problem for the dynamic system (1)–(4), (6).

Problem 1. For a fixed period of time $\overline{\vartheta, T} \subseteq \overline{0,T}$ ($\vartheta < T$, $T \in \mathbb{N}$) and implementation ϑ-position $w(\vartheta) = \{\vartheta, \overline{x}(\vartheta)\} \in \hat{\boldsymbol{W}}(\vartheta)$ ($w(0) = w_0$) of manager P in the dynamic system (1)–(4), (6) it is required to find a set $\overline{\boldsymbol{U}}^{(e)}_{\Phi_{\overline{\vartheta,T}}}(\overline{\vartheta,T}, w(\vartheta)) \subseteq \overline{\boldsymbol{U}}(\overline{\vartheta,T}, w(\vartheta))$ of optimal program controls by the number of employees and the sales system of the Retail Unit of the Bank $\overline{u}^{(e)}(\cdot) \in \overline{\boldsymbol{U}}(\overline{\vartheta,T}, w(\vartheta))$ of manager P, which is determined by the relation:

$$\begin{gathered}\overline{\boldsymbol{U}}^{(e)}_{\Phi_{\overline{\vartheta,T}}}(\overline{\vartheta,T},w(\vartheta))=\{\overline{u}^{(e)}(\cdot):\overline{u}^{(e)}(\cdot)\in\overline{\boldsymbol{U}}(\overline{\vartheta,T},w(\vartheta)),\\ \boldsymbol{\Phi}^{(e)}_{\overline{\vartheta,T}}=\boldsymbol{\Phi}_{\overline{\vartheta,T}}(w(\vartheta),\overline{u}^{(e)}(\cdot))=\max_{\overline{u}(\cdot)\in\overline{\mathbf{U}}(\overline{\vartheta,T},w(\vartheta))}\boldsymbol{\Phi}_{\overline{\vartheta,T}}(w(\vartheta),\overline{u}(\cdot))=\\ =\max_{\overline{u}(\cdot)\in\overline{\mathbf{U}}(\overline{\vartheta,T},w(\vartheta))}\boldsymbol{F}_{\overline{\vartheta,T}}(\varphi_{\overline{\vartheta,T}}(T;\overline{x}(\vartheta),\overline{u}(\cdot)))=\boldsymbol{F}_{\overline{\vartheta,T}}(\overline{x}^{(e)}_{\overline{\vartheta,T}}(T))=\boldsymbol{F}^{(e)}_{\overline{\vartheta,T}}=c^{(e)}_{\Phi_{\overline{\vartheta,T}}}(\overline{\vartheta,T},w(\vartheta))\},\end{gathered}\tag{7}$$

as an implementation of a finite sequence of one-step operations only.

Here the functional $\boldsymbol{\Phi}_{\overline{\vartheta,T}}$ is defined by the relation (6).

The number $\boldsymbol{\Phi}^{(e)}_{\overline{\vartheta,T}}=c^{(e)}_{\Phi_{\overline{\vartheta,T}}}(\overline{\vartheta,T},w(\vartheta))=\boldsymbol{F}^{(e)}_{\overline{\vartheta,T}}$ will be called the optimal value of the result of the process of the program control of the manager P on the time interval $\overline{\vartheta,T}$ for the discrete dynamic system (1)–(4), (6) relative to its ϑ-position $w(\vartheta)$ and functional $\boldsymbol{\Phi}_{\overline{\vartheta,T}}$.

Note that the solution to Problem 1, defined by relation(7), exists [9] and in [8] a constructive general scheme for finding it is given, which can be briefly described as an implementation of the following sequence of actions.

Then we can formulate the following nonlinear multi-step problem of optimal adaptive control of the number and sales system of the Bank's Retail Unit within the discrete dynamic system (1)–(4), (6).

Problem 2. For a given time interval $\overline{0,T}$ ($T\in\mathbb{N}$) and initial position $w_0=\{0,\overline{x}_0\}\in\hat{\boldsymbol{W}}_0$ of the manager P in a discrete dynamic system (1)–(4), (6) $\overline{\boldsymbol{U}}^*_a$ — the set of permissible strategies of adaptive control. Then it is required to find a valid strategy for optimal adaptive control of the number of employees and the sales system of the Bank's Retail Unit $\overline{\boldsymbol{U}}^{(e)}_a=\overline{\boldsymbol{U}}^{(e)}_a(w(\vartheta))\in\overline{\boldsymbol{U}}^*_a$, $w(\vartheta)=\{\vartheta,\overline{x}(\vartheta)\}\in\hat{\boldsymbol{W}}(\vartheta),\vartheta\in\overline{0,T-1}$ manager P, which is defined as follows :

1) $\forall w^{(e)}(\vartheta)=\{\vartheta,\overline{x}^{(e)}(\vartheta)\}\in\hat{\boldsymbol{W}}(\vartheta),\ \vartheta\in\overline{0,T-1}$, relies

$$\overline{\boldsymbol{U}}^{(e)}_a=\overline{\boldsymbol{U}}^{(e)}_a(w^{(e)}(\vartheta)),\tag{8}$$

where the set $\overline{\boldsymbol{U}}^{(e)}_a(w^{(e)}(\vartheta))$ is determined by

$$\begin{gathered}\overline{\boldsymbol{U}}^{(e)}_a(w^{(e)}(\vartheta))=\{\overline{u}^{(e)}(\vartheta):\ \overline{u}^{(e)}(\vartheta)=(u^{(e)}(\tau_\vartheta),A(\tau_\vartheta))\in\overline{\boldsymbol{U}}_1(\vartheta,y^{(e)}(\tau_\vartheta),y^{(e)}(\tau_\vartheta-1)),\\ \overline{u}^{(e)}(\vartheta)=\overline{u}^{(e)}_*(\vartheta),\overline{u}^{(e)}_*(\vartheta)\in\overline{\boldsymbol{U}}^{(e)}_{\Phi_{\overline{\vartheta,T}}}(\overline{\vartheta,T},w^{(e)}(\vartheta))\},\end{gathered}\tag{9}$$

and $w^{(e)}(\vartheta)=\{\vartheta,\overline{x}^{(e)}(\vartheta)\}=(y^{(e)}(\vartheta),x^{(e)}(\vartheta))=(y^{(e)}(\tau_\vartheta),x^{(e)}(\vartheta))$, $\overline{x}^{(e)}(\vartheta)=\varphi_{\overline{0,\vartheta}}(\vartheta;\overline{x}_0,\overline{u}^{(e)}_{a,\vartheta}(\cdot))$, $\overline{u}^{(e)}_{a,\vartheta}(\cdot)\in\overline{\mathbf{U}}(\overline{0,\vartheta},w_0)$, and control action $\overline{u}^{(e)}_{a,\vartheta}(\cdot)$ is generated by the implementation of the strategy $\overline{\boldsymbol{U}}^{(e)}_a$ on the time interval $\overline{0,\vartheta}$;

2) $\forall w_*(\vartheta)=\{\vartheta,\overline{x}_*(\vartheta)\}\in\{\hat{\boldsymbol{W}}(\vartheta)\setminus w^{(e)}(\vartheta)\}$, $\vartheta\in\overline{0,T-1}$, relies according to (2)

$$\overline{\boldsymbol{U}}^{(e)}_a=\overline{\boldsymbol{U}}^{(e)}_a(w_*(\vartheta))=\overline{\boldsymbol{U}}_1(\vartheta,y_*(\tau_\vartheta),y_*(\tau_\vartheta-1)),\tag{10}$$

as an implementation of a finite sequence of only one-step operations (here $w_*(\vartheta)=\{\vartheta,\overline{x}_*(\vartheta)\}=(y_*(\vartheta),x_*(\vartheta))=(y_*(\tau_\vartheta),x_*(\vartheta))$).

Let the phase trajectory of system (1)–(4) $\overline{x}^{(e)}_a(\cdot)=\{\overline{x}^{(e)}_a(t)\}_{t\in\overline{0,T}}=\varphi_{\overline{0,T}}(\cdot;\overline{x}_0,\overline{u}^{(e)}_{a,T}(\cdot))$, $\overline{u}^{(e)}_{a,T}(\cdot)\in\overline{\mathbf{U}}(\overline{0,T},w_0)$ is generated by the implementation of the strategy $\overline{\boldsymbol{U}}^{(e)}_a\in\overline{\boldsymbol{U}}^*_a$ on the time interval $\overline{0,T}$, and the $(T-1)$-position of the manager P is such that $w^{(e)}_a(T-1)=\{T-1,x^{(e)}_a(T-1)\}$. Then the number $\boldsymbol{\Phi}^{(e,a)}_{\overline{0,T}}=c^{(e,a)}_{\Phi_{\overline{0,T}}}(\overline{0,T},w_0)=\boldsymbol{F}^{(e,a)}_{\overline{0,T}}$, which is determined on the basis of (7) and the following relationship:

$$\begin{gathered}\boldsymbol{\Phi}^{(e,a)}_{\overline{0,T}}=c^{(e,a)}_{\Phi_{\overline{0,T}}}(\overline{0,T},w_0)=\boldsymbol{F}^{(e,a)}_{\overline{0,T}}=c^{(e)}_{\Phi_{\overline{T-1,T}}}(\overline{T-1,T},w^{(e)}_a(T-1))=\\ =\boldsymbol{F}^{(e)}_{\overline{T-1,T}}=\max_{\overline{u}(T-1)\in\overline{\boldsymbol{U}}(\overline{T-1,T},w^{(e)}_a(T-1))}\boldsymbol{F}_{\overline{T-1,T}}(\varphi_{\overline{T-1,T}}(T;\overline{x}(T-1),\overline{u}(T-1))),\end{gathered}\tag{11}$$

we will call the optimal value of the result of the process of adaptive control of the manager P on the time interval $\overline{0,T}$ for the discrete dynamic system (1)–(4),(6) with respect to its initial position w_0 and functional $\boldsymbol{\Phi}_{\overline{0,T}}$.

The ratio, similar to (11), determines the optimal value of the result of the process of adaptive control of the manager P and at any time interval $\overline{\vartheta,T}\subset\overline{0,T}$. Note that among the conditions described above for the parameters of the system (1)–(4),(6), the solution to this problem exists [9]and in the next part of this work we will present a constructive general scheme for finding it.

Note that Problem 2 is fundamental in this work, but its formalization and solution are based on Problem 1, an auxiliary problem of optimal program control.

THE GENERAL SCHEME FOR SOLVING PROBLEM 2

We present a general scheme for solving Problem 2, which is based on the results of [8],[9] and the general scheme for solving Problem 1 described above.

1. The formation of a set of initial data that completely describes the system of equations (1), constraints (2)–(4) and the objective function of the form (6) is carried out. Relies $\vartheta = 0$ and $w_a^{(e)}(\vartheta) = \{\vartheta, \overline{x}_a^{(e)}(\vartheta)\} \in \hat{\boldsymbol{W}}(\vartheta)$ where $\overline{x}_a^{(e)}(\vartheta) = \overline{x}(0) = \{y_0, x_0\} \in \overline{\boldsymbol{X}}_1(0)$.

Then the problem of optimizing the complex program control of the number of employees of the Bank's Retail Unit and their sales system can be formulated as follows.

2. The real implementation $w_*(\vartheta) = \{\vartheta, \overline{x}_*(\vartheta)\} \in \boldsymbol{W}(\vartheta)$ of the ϑ-position of the manager P is measured . Then if $(w_*(\vartheta) \neq w_a^{(e)}(\vartheta)) \wedge (w_*^{(e)}(\vartheta) \notin \boldsymbol{W}(\vartheta))$, then it is assumed $w_a^{(e)}(\vartheta) = w_*(\vartheta)$, $\widetilde{\boldsymbol{U}}_a^{(e)}(w_a^{(e)}(\vartheta)) = \overline{\boldsymbol{U}}_1(\vartheta, y_a^{(e)}(\tau_\vartheta), y_a^{(e)}(\tau_\vartheta - 1))$ and the transition to point 7 of this scheme is carried out; if $(w_*(\vartheta) \neq w_a^{(e)}(\vartheta)) \wedge (w_*^{(e)}(\vartheta) \in \boldsymbol{W}(\vartheta))$, then it is assumed $w_a^{(e)}(\vartheta) = w_*(\vartheta)$ and the following item of the scheme is executed (here $w_a^{(e)}(\vartheta) = \{\vartheta, \overline{x}_a^{(e)}(\vartheta)\}, \overline{x}_a^{(e)}(\vartheta) = (y_a^{(e)}(\tau_\vartheta), x_a^{(e)}(\vartheta))$).

3. On the basis of formulas (2), (5) a finite set of admissible over the period of time $\overline{\vartheta, T}$ program controls $\overline{\boldsymbol{U}}(\overline{\vartheta, T}, w_a^{(e)}(\vartheta)) = \{\overline{u}^{(k)}(\cdot)\}_{k \in \overline{1, K_\vartheta}} = \{(u^{(k)}(\cdot), A^{(k)}(\cdot))\}_{k \in \overline{1, K_\vartheta}} = \{(\{u^{(k)}(\tau_t)\}_{t \in \overline{\vartheta, T-1}}, \{A^{(k)}(\tau_t)\}_{t \in \overline{\vartheta, T-1}})\}_{k \in \overline{1, K_\vartheta}} \subset \mathbb{R}^{\overline{p} \times (T-\vartheta)}$, of the manager P, consisting of the $K_\vartheta = (N_{\tau_t} \times M_{\tau_t})^{(T-\vartheta)}$ elements, $K_\vartheta \in \mathbb{N}$ ($\overline{p} = m(n+1) \in \mathbb{N}$), is formed.

4. On the basis of formulas (1), (6), for each $k \in \overline{1, K_\vartheta}$ and the corresponding program control allowed for the period of time $\overline{\vartheta, T}$ $\overline{u}^{(k)}(\cdot) \in \overline{\boldsymbol{U}}(\overline{\vartheta, T}, w_a^{(e)}(\vartheta))$ of the manager P, the number $\boldsymbol{\Phi}_{\overline{\vartheta, T}}^{(k)} = \boldsymbol{\Phi}_{\overline{\vartheta, T}}(w_a^{(e)}(\vartheta), \overline{u}^{(k)}(\cdot)) = p^{(k)}(T)$ is calculated and the pair $(\overline{u}^{(k)}(\cdot), \boldsymbol{\Phi}_{\overline{\vartheta, T}}^{(k)})$ is stored.

5. From the solution of a finite discrete optimization problem, in accordance with relation (7), the set $\widetilde{\boldsymbol{U}}_{\boldsymbol{\Phi}_{\overline{\vartheta,T}}}^{(e)}(\overline{\vartheta, T}, w_a^{(e)}(\vartheta))$ of sets $\{\overline{u}^{(k)^{(e)}}(\cdot), \boldsymbol{\Phi}_{\overline{\vartheta, T}}^{(k,e)}\} \in \overline{\boldsymbol{U}}(\overline{\vartheta, T}, w_a^{(e)}(\vartheta)) \times \mathbb{R}^1$, which is formed from the solution of the following finite discrete optimization problem:

$$\widetilde{\boldsymbol{U}}_{\boldsymbol{\Phi}_{\overline{\vartheta,T}}}^{(e)}(\overline{\vartheta, T}, w(\vartheta)) = \{\overline{u}^{(k,e)}(\cdot) : \ \overline{u}^{(k,e)}(\cdot) \in \overline{\boldsymbol{U}}(\overline{\vartheta, T}, w_a^{(e)}(\vartheta)),$$
$$\boldsymbol{\Phi}_{\overline{\vartheta, T}}^{(k,e)} = \boldsymbol{\Phi}_{\overline{\vartheta, T}}^{(k,e)}(w_a^{(e)}(\vartheta)), \overline{u}^{(k,e)}(\cdot)) = \max_{k \in \overline{1, K_\vartheta}} \boldsymbol{\Phi}_{\overline{\vartheta, T}}(w_a^{(e)}(\vartheta)), \overline{u}^{(k)}(\cdot)) =$$
$$= \max_{k \in \overline{1, K_\vartheta}} \boldsymbol{F}_{\overline{\vartheta, T}}(\varphi_{\overline{\vartheta, T}}(T, \overline{x}_a^{(e)}(\vartheta)), \overline{u}^{(k)}(\cdot)) = \boldsymbol{F}_{\overline{\vartheta, T}}^{(k,e)} = c_{\boldsymbol{\Phi}_{\overline{\vartheta,T}}}^{(k,e)}(\overline{\vartheta, T}, w_a^{(e)}(\vartheta))\},$$

is computed.

6. The following non-empty set is formed:

$$\begin{aligned} \widetilde{\boldsymbol{U}}_{\boldsymbol{\Phi}}^{(e)}(w_a^{(e)}(\vartheta)) = \{\widetilde{u}_a^{(e)}(\vartheta) : \widetilde{u}_a^{(e)}(\vartheta) = (u_a^{(e)}(\tau_\vartheta), A_a^{(e)}(\tau_\vartheta)) \in \overline{\boldsymbol{U}}_1(\vartheta, y_a^{(e)}(\tau_\vartheta), y_a^{(e)}(\tau_\vartheta - 1)), \\ \widetilde{u}_a^{(e)}(\vartheta) = \overline{u}^{(k^{(e)})}(\vartheta), \overline{u}^{(k^{(e)})}(\cdot) \in \widetilde{\boldsymbol{U}}_{\boldsymbol{\Phi}_{\overline{\vartheta,T}}}^{(e)}(\overline{\vartheta, T}, w_a^{(e)}(\vartheta))\}, \end{aligned} \tag{12}$$

where $w_a^{(e)}(\vartheta) = \{\vartheta, \overline{x}_a^{(e)}(\vartheta)\}, \overline{x}_a^{(e)}(\vartheta) = (y_a^{(e)}(\vartheta), x_a^{(e)}(\vartheta)) = (y_a^{(e)}(\tau_\vartheta), x_a^{(e)}(\vartheta))$.

7. Based on (2), (9) any control $\widetilde{u}_a^{(e)}(\vartheta) \in \widetilde{\boldsymbol{U}}^{(e)}(w_a^{(e)}(\vartheta))$ is selected that corresponds to a time period ϑ.

8. Calculates the $(\vartheta + 1)$-position $w_a^{(e)}(\vartheta + 1) = \{\vartheta + 1, \overline{x}_a^{(e)}(\vartheta + 1)\} \in \boldsymbol{W}(\vartheta, w_a^{(e)}(\vartheta), \vartheta + 1, \widetilde{u}_a^{(e)}(\vartheta)) \subset \hat{\boldsymbol{W}}(\vartheta + 1)$ of the manager P, where $\overline{x}_a^{(e)}(\vartheta + 1) = \varphi_{\overline{\vartheta, \vartheta+1}}(\vartheta + 1; \overline{x}_a^{(e)}(\vartheta), \widetilde{u}_a^{(e)}(\vartheta)) \in \overline{\boldsymbol{X}}_1(\vartheta + 1)$.

9. Rely $\vartheta := \vartheta + 1$. Then if $\vartheta < T$, then the transition to point 2 of this scheme is in progress; otherwise, i.e. as $\vartheta = T$, the transition to the next item of the scheme is in progress.

10. The admissible strategy of adaptive control $\widetilde{\boldsymbol{U}}_a^{(e)} = \widetilde{\boldsymbol{U}}_a^{(e)}(w_a^{(e)}(\vartheta)) \in \overline{\boldsymbol{U}}_a^*$, $w_a^{(e)}(\vartheta) = \{\vartheta, \overline{x}_a^{(e)}(\vartheta)\} \in \hat{\boldsymbol{W}}(\vartheta)$, $\vartheta \in \overline{0, T-1}$ manager P, which is described above, was formed as follows:

1) $\forall w_a^{(e)}(\vartheta) = \{\vartheta, \overline{x}_a^{(e)}(\vartheta)\} \in \hat{\boldsymbol{W}}(\vartheta)$, $\vartheta \in \overline{0, T-1}$ relies

$$\widetilde{\boldsymbol{U}}_a^{(e)} = \widetilde{\boldsymbol{U}}_a^{(e)}(w_a^{(e)}(\vartheta)), \tag{13}$$

where the set $\widetilde{\boldsymbol{U}}_a^{(e)}(w_a^{(e)}(\vartheta))$ is determined according to (8), and (9);

2) $\forall w_*(\vartheta) = \{\vartheta, \overline{x}_*(\vartheta)\} \notin \hat{\boldsymbol{W}}(\vartheta)$, $\vartheta \in \overline{0, T-1}$, according to (10) relies

$$\overline{\boldsymbol{U}}_a^{(e)} = \overline{\boldsymbol{U}}_a^{(e)}(w_*(\vartheta)) = \overline{\boldsymbol{U}}_1(\vartheta, y_*(\tau_\vartheta), y_*(\tau_\vartheta - 1)). \tag{14}$$

Then, on the basis of the results of [8],[9], it can be shown that for the admissible adaptive control strategy $\widetilde{\boldsymbol{U}}_a^{(e)} = \widetilde{\boldsymbol{U}}_a^{(e)}(w_a^{(e)}(\vartheta)) \in \overline{\boldsymbol{U}}_a^*, w_a^{(e)}(\vartheta) = \{\vartheta, \overline{x}_a^{(e)}(\vartheta)\} \in \hat{\boldsymbol{W}}(\vartheta), \vartheta \in \overline{0, T-1}$ of the manager P, implemented by the above, the following relations are true:

$$\begin{gathered}\overline{\boldsymbol{U}}_a^{(e)} = \widetilde{\boldsymbol{U}}_a^{(e)} = \widetilde{\boldsymbol{U}}_a^{(e)}(w_a^{(e)}(\vartheta)) \in \overline{\boldsymbol{U}}_a^*, w_a^{(e)}(\vartheta) = \{\vartheta, \overline{x}_a^{(e)}(\vartheta)\} \in \hat{\boldsymbol{W}}(\vartheta), \vartheta \in \overline{0, T-1};\\ \boldsymbol{\Phi}_{\overline{0,T}}^{(e,a)} = c_{\boldsymbol{\Phi}_{\overline{0,T}}}^{(e,a)}(\overline{0, T}, w_0) = \widetilde{c}_{\boldsymbol{\Phi}_{\overline{0,T}}}^{(e,a)}(\overline{0, T}, w_0) = \widetilde{\boldsymbol{\Phi}}_{\overline{0,T}}^{(e,a)},\end{gathered} \tag{15}$$

where the number $\widetilde{c}_{\boldsymbol{\Phi}_{\overline{0,T}}}^{(e,a)}(\overline{0, T}, w_0)$ is calculated according to (11) when implemented for a period of time $\overline{0, T}$ the control $\widetilde{u}_a^{(e)}(\cdot) = \{\widetilde{u}_a^{(e)}(\vartheta)\}_{\vartheta \in \overline{0,T-1}}$ of manager P, corresponding to the implementation of the strategy $\widetilde{\boldsymbol{U}}_a^{(e)} \in \overline{\boldsymbol{U}}_a^*$, i.e. according to (12)–(14) $\forall \vartheta \in \overline{0, T-1}: \ \widetilde{u}_a^{(e)}(\vartheta) \in \widetilde{\boldsymbol{U}}_a^{(e)}(w_a^{(e)}(\vartheta))$.

From relations (15) it follows that the strategy $\widetilde{\boldsymbol{U}}_a^{(e)} \in \overline{\boldsymbol{U}}_a^*$ formed as a result of the implementation of this scheme is the strategy of optimal adaptive control of the number of employees and the sales system of the Bank's Retail Unit, and the number $\widetilde{\boldsymbol{\Phi}}_{\overline{0,T}}^{(e,a)} = \widetilde{c}_{\boldsymbol{\Phi}_{\overline{0,T}}}^{(e,a)}(\overline{0, T}, w_0)$ that satisfies relations (8)–(11) form the solution to Problem 2.

From the general scheme for solving Problem 2, described above, relations (8)–(15) and the results of [8],[9] the next statement follows, which is the main result of this paper.

Statement. For a fixed period of time $\overline{0, T}$ ($\vartheta \leq T, T \in \mathbb{N}$) and the initial position $w(0) = \{0, \overline{x}(0)\} = \{0, \overline{x}_0\} \in \hat{\boldsymbol{W}}_0 = \hat{\boldsymbol{W}}(0)$ of the manager P in the discrete dynamic system (1)–(4), (6), for Problem 2 formed as a result of implementation of the general solution scheme, its control strategy $\widetilde{\boldsymbol{U}}_a^{(e)} \in \overline{\boldsymbol{U}}_a^*$ is determined by relations (12)-(14), is the strategy of optimal adaptive control of the number of employees and the sales system of the Bank's Retail Unit for the Problem 2, i.e. $\overline{\boldsymbol{U}}_a^{(e)} = \widetilde{\boldsymbol{U}}_a^{(e)} = \widetilde{\boldsymbol{U}}_a^{(e)}(w_a^{(e)}(\vartheta)) \in \overline{\boldsymbol{U}}_a^*$, $w_a^{(e)}(\vartheta) = \{\vartheta, \overline{x}_a^{(e)}(\vartheta)\} \in \hat{\boldsymbol{W}}(\vartheta)$, $\vartheta \in \overline{0, T-1}$, and the number $\widetilde{c}_{\boldsymbol{\Phi}_{\overline{0,T}}}^{(e,a)}(\overline{0, T}, w_0)$ is the optimal value of the result of the process of the adaptive terminal control of the manager P on the time interval $\overline{0, T}$ for this promlem, i.e. $\boldsymbol{\Phi}_{\overline{0,T}}^{(e,a)} = c_{\boldsymbol{\Phi}_{\overline{0,T}}}^{(e,a)}(\overline{0, T}, w_0) = \widetilde{c}_{\boldsymbol{\Phi}_{\overline{0,T}}}^{(e,a)}(\overline{0, T}, w_0) = \widetilde{\boldsymbol{\Phi}}_{\overline{0,T}}^{(e,a)}$, and the data set $(\widetilde{u}_a^{(e)}(\cdot), \overline{x}_a^{(e)}(\cdot), \widetilde{\boldsymbol{\Phi}}_{\overline{0,T}}^{(e,a)}) \in \mathbb{R}^{\overline{n} \times T} \times \mathbb{R}^{\overline{n} \times (T+1)} \times \mathbb{R}^1$ formed as a result of the implementation of the strategy $\widetilde{\boldsymbol{U}}_a^{(e)}$ is the output result of solving Problem 2. Moreover, all elements of this set are calculated by implementing a finite number of only one-step algebraic operations and finding solutions to finite discrete optimization problems.

From this statement it follows that on the basis of the developed formalized description of the problem of optimal adaptive control of the number of employees and the sales system of the Bank's Retail Unit, as well as a constructive general solution for the formulated corresponding Problem 2, it is possible to develop numerical algorithms and implement computer simulation of the search for solving this problem. One of the variants of the developed numerical method for solving Problem 2 is described in next section of this article.

NUMERICAL ALGORITHM FOR SOLVING THE PROBLEM OF OPTIMAL ADAPTIVE CONTROL

Consider the numerical algorithm for solving the formulated Problem 2 — problem of optimal adaptive control by the number of employees of the Bank's Retail Unit and their sales system with vector objective function on a concrete practical example, which is a special case of the general economic-mathematical model (1)–(4), (6).

Let $n = 8, m = 7$ and the parameters of the system (1)–(4) be described on the time interval $\overline{0, T}$ in the time period t ($t \in \overline{0, T}$) by the following data:

$\tau = 6$, i.e. the number of employees of the bank can vary with a periodicity of 6 months - no more than once every six months;

$x(t) = (x_1(t), x_2(t), \ldots, x_8(t))' \in \mathbb{R}^8$;

$y(t) = (y_1(t), y_2(t), \ldots, y_7(t))' \in \mathbb{R}^7$.

We know the initial state at time $t = 0$ of the phase vector $\overline{x}(0) = \{y(0), x(0)\}' \in \mathbb{R}^{15}$.

It is assumed that the restriction of (2) to the discrete control value $u(t) = (u_1(t), u_2(t), \ldots, u_7(t))' \in \mathbb{R}^7$ for each $t \in \overline{0, T-1}$ has the following concrete form:

$$\overline{u}(t) = (u(\tau_t), A(\tau_t)) \in \overline{\boldsymbol{U}}_1(t, y(\tau_t), y(\tau_t - 1)) = (\boldsymbol{U}_1(\tau_t) \times \mathbf{A}_1(\tau_t)) \subset \mathbb{R}^{\overline{p}} (\overline{p} = 7(8+1) = 63),$$

$$u(\tau_t) \in \boldsymbol{U}_1(\tau_t) \subset \mathbb{R}^7, t \in \overline{0, T-1},$$

$$A(\tau_t) \in \mathbf{A}_1(\tau_t) \subset \mathbb{R}^{168}.$$

In addition, the control $u(t)$ determines the first component of the generalized control action $\overline{u}(t) = \{u(t), A(t)\} \in \overline{\mathbf{U}}(t)$, where $u(t) = u^{(k)}(t) \in \mathbf{U}_1(t)$ for all $t \in \overline{0, T-1}$ and $k \in \overline{1, 3}$, and it is assumed that for each $k \in \overline{1, 3}$ a single initial control value $u(0) = u^{(k)}(0) = \mathbf{0}_7 \in \mathbf{U}_1(0)$ is given, i.e. $\mathbf{U}_1(0) = \{u(0)\}$ is a one-element set, where $\mathbf{0}_7 = (0, 0, \ldots, 0)'$ is the zero vector of the space $\mathbb{R}^7$.

In accordance with the restriction (2), we also define a sequence of matrices $A_1(t) = \{A(t) : A(t) \in \{A^{(1)}(t), A^{(2)}(t), A^{(3)}(t)\}\}$, consisting of only three matrices, such that for each time period t ($t \in \overline{0, T-1}$), the corresponding matrix $A^{(k)}(t) \in A_1(t)$ ($k \in \overline{1, 3}$) defines the second component of the generalized control action $\overline{u}(t) = \{u(t), A(t)\} \in \overline{\mathbf{U}}(t)$, where $A(t) = A^{(k)}(t)$.

This means that you can simulate the process of program control under different matrices. It is assumed that for each $k \in \overline{1, 3}$ a single initial value of the sales norm matrix $A(0) = A^{(k)}(0) \in A_1(0)$ is given.

The restriction of (3), (4) has the following concrete form:

$$\overline{x}(t) = (y(t), x(t)) \in \overline{\boldsymbol{X}}_1(t) = \boldsymbol{Y}_1(t) \times \boldsymbol{X}_1(t),$$

$$y(t) \in \boldsymbol{Y}_1(t), x(t) \in \boldsymbol{X}_1(t),$$

$$\boldsymbol{Y}_1(t) = \{y(t) : y(t) = (y_1(t), y_2(t), \ldots, y_7(t))' \in \mathbb{R}^7, \forall i \in \overline{1, 7} : y_i(t) \geq 0\},$$

$$\boldsymbol{X}_1(t) = \{x(t) : x(t) = (x_1(t), x_2(t), \ldots, x_8(t))' \in \mathbb{R}^8, \forall i \in \overline{1, 8} : x_i(t)) \geq 0\}.$$

Then, the numerical algorithm for modeling the solution of the Problem 2 of optimal adaptive control for the number of employees and sales system of a Bank's Retail Unit in the presence of a vector objective function of the form (6), based on discribed in the previous part general scheme of the solving this problem and Statement, can be represented as the implementation of the following sequence of actions.

Step 0. Forming the initial data.

0.1. A natural number $T \in \mathbb{N}$ is introduced, which determines the period of optimization of the control of the process under consideration; it is assumed that $n = 8, m = 7$.

0.2. For $t = 0$ the initial value of the phase vector $\overline{x}(0) = \{y(0), x(0)\}' \in \mathbb{R}^{15}$ is formed.

0.3. For $t = 0$ we form a vector $u(0) = (u_1(0), u_2(0), \ldots, u_7(0))' \in \mathbb{R}^7$ which, in accordance with the constraint (2), defines a one-element set $\mathbf{U}_1(0, y(\tau_0), y(\tau_0 - 1)) = \{u(0)\}$.

0.4. For $t = 0$ a matrix $A(0)$ is formed, which, in accordance with the constraint (2), defines a singleton set $A_1(0) = \{A(0)\}$.

0.5. Sets $\overline{\mathbf{X}}_1(w), w \in \overline{0, T}$, which discribe description (3), are formed.

0.6. It is assumed that $w = 0$ and $w_a^{(e)}(\vartheta) = \{\vartheta, \overline{x}_a^{(e)}(\vartheta)\} \in \hat{\boldsymbol{W}}(\vartheta)$, where $\overline{x}_a^{(e)}(w) = \overline{x}(0) = \{y_0, x_0\} \in \overline{\mathbf{X}}_1(0)$.

0.7. The items of matrix $A^{(1)}(\tau_\vartheta)$, $A^{(2)}(\tau_\vartheta)$ and $A^{(3)}(\tau_\vartheta)$ are formed.

Step 1. Measurement of the real phase vector of the system.

1.1. The beginning of cycle 1 with integer variable $\vartheta \in \overline{0, T-1}$.

1.2. The real implementation $w_*(\vartheta) = \{\vartheta, \overline{x}_*(\vartheta)\} \in \boldsymbol{W}(\vartheta)$ by the ϑ-position of the manager P is measured. Then if $(w_*(\vartheta) \neq w_a^{(e)}(\vartheta)) \wedge (w_*(\vartheta) \notin \boldsymbol{W}(\vartheta))$, then it is assumed that $w_a^{(e)}(\vartheta) = w_*(\vartheta)$, a set $\widetilde{\overline{\boldsymbol{U}}}_a^{(e)}(w_a^{(e)}(\vartheta)) = \overline{\boldsymbol{U}}_1(\vartheta, y_a^{(e)}(\tau_\vartheta), y_a^{(e)}(\tau_\vartheta - 1))$ is formed and the transition to point 3.7 of this algorithm is carried out; if $(w_*(\vartheta) \neq w_a^{(e)}(\vartheta)) \wedge (w_*(\vartheta) \in \boldsymbol{W}(\vartheta))$, then $w_a^{(e)}(\vartheta) = w_*(\vartheta)$ is assumed and the next item of the algorithm is executed (here $w_a^{(e)}(\vartheta) = \{\vartheta, \overline{x}_a^{(e)}(\vartheta)\}$, $\overline{x}_a^{(e)}(\vartheta) = (y_a^{(e)}(\tau_\vartheta), \overline{x}_a^{(e)}(\vartheta))$).

Step 2. Formation of a set of admissible program control actions.

2.1. On the basis of formulas (2)–(4), (6), a finite set $\overline{\boldsymbol{U}}(\overline{\vartheta, T}, w_a^{(e)}(\vartheta))$ of admissible on the time interval $\overline{\vartheta, T}$ program controls of the manager P, corresponding to its ϑ-position $w_a^{(e)}(\vartheta) = \{\vartheta, \overline{x}_a^{(e)}(\vartheta)\} \in \hat{\boldsymbol{W}}(\vartheta)$, is formed as follows:

$$\overline{\boldsymbol{U}}(\overline{\vartheta, T}, w_a^{(e)}(\vartheta)) = \{\overline{u}^{(k)}(\cdot) : \overline{u}^{(k)}(\cdot) = (u^{(k)}(\cdot), A^{(k)}(\cdot)) =$$

$$= (\{u^{(k)}(\tau_t)\}_{t \in \overline{\vartheta, T-1}}, \{A^{(k)}(\tau_t)\}_{t \in \overline{\vartheta, T-1}}) \in \mathbb{R}^{\overline{p} \times (T - \vartheta)}, k \in \overline{1, K_\vartheta},$$

$$u(\tau_t) \in \{u^{(1)}(\tau_t), u^{(2)}(\tau_t), u^{(3)}(\tau_t)\} \subset \mathbb{R}^7,$$

$$A(\tau_t) \in \{A^{(1)}(\tau_t), A^{(2)}(\tau_t), A^{(3)}(\tau_t)\} \subset \mathbb{R}^{168},$$

where $K_\vartheta = 9^{(T-\vartheta-1)}$- the number of elements of $\overline{u}^{(k)}(\cdot) = \{u^{(k)}(\tau_t)\}_{t\in\overline{\vartheta,T-1}}, k \in \overline{1,K_\vartheta}$, of set $\overline{\boldsymbol{U}}(\overline{\vartheta,T}, w_a^{(e)}(\vartheta))$; $\overline{p} = 7(8+1) = 63$.

Step 3. Formation of the solution of the auxiliary problem of optimal program control.

3.1. A real numerical value $\mathbf{F}^{(e)}(T) = -10^{10}$ and set $\widetilde{\boldsymbol{U}}^{(e)}(w_a^{(e)}(\vartheta)) = \emptyset$ are formed.

3.2. Start of cycle 2 in integer variable $k \in \overline{1,K_\vartheta}$.

3.3. The control action $\overline{u}^{(k)}(\cdot) = \{u^{(k)}(\cdot), A^{(k)}(\cdot)\} \in \overline{\mathbf{U}}(\overline{\vartheta,T}, w_a^{(e)}(\vartheta))$ of the manager P at the time interval $\overline{\vartheta,T}$ is formed, where $\overline{u}^{(k)}(0) = \{u(0), A(0)\}$.

3.3. Based on the formulas (1), (11) for the program control $\overline{u}^{(k)}(\cdot)$ of the manager P, the number $\boldsymbol{F}^{(k)}_{\overline{\vartheta,T}} = \boldsymbol{F}_{\overline{\vartheta,T}}(\overline{x}^{(k)}(T)) = \mathbf{\Phi}_{\overline{\vartheta,T}}(w_a^{(e)}(\vartheta), \overline{u}^{(k)}(\cdot)) = \mathbf{\Phi}^{(k)}_{\overline{\vartheta,T}}$ is calculated and the pair $(\overline{u}^{(k)}(\cdot), \mathbf{\Phi}^{(k)}_{\overline{\vartheta,T}})$ is stored, where $\overline{x}^{(k)}(T) = \varphi_{\overline{\vartheta,T}}(T, \overline{\vartheta,T}, \overline{x}(\vartheta), \overline{u}^{(k)}(\cdot))$.

3.4. If $\boldsymbol{F}^{(k)}_{\overline{\vartheta,T}} = \boldsymbol{F}_{\overline{\vartheta,T}}(\overline{x}^{(k)}(T)) \neq \boldsymbol{F}^{(e)}(T)$ then, $\widetilde{\boldsymbol{U}}^{(e)}(w_a^{(e)}(\vartheta)) = \emptyset$ is assumed and the transition to item 3.5. is carried out; if $\boldsymbol{F}^{(k)}_{\overline{\vartheta,T}} = \boldsymbol{F}_{\overline{\vartheta,T}}(\overline{x}^{(k)}(T)) = \boldsymbol{F}^{(e)}(T)$, then the operation $\widetilde{\boldsymbol{U}}^{(e)}(w_a^{(e)}(\vartheta)) = \widetilde{\boldsymbol{U}}^{(e)}(w_a^{(e)}(\vartheta)) \bigcup \{\widetilde{u}_a^{(e)}(\vartheta)\}$, is performed, where $\widetilde{u}_a^{(e)}(\vartheta) = \overline{u}^{(k)}(\vartheta)$ and the transition to item 3.6 is carried out; otherwise, it goes to point 3.6.

3.5. The value ofa real variable $\boldsymbol{F}^{(e)}(T) = \boldsymbol{F}^{(k)}_{\overline{\vartheta,T}}$ and the set $\widetilde{\boldsymbol{U}}^{(e)}(w_a^{(e)}(\vartheta)) = \widetilde{\boldsymbol{U}}^{(e)}(w_a^{(e)}(\vartheta)) \bigcup \{\widetilde{u}_a^{(e)}(\vartheta)\}$, are formed, where $\widetilde{u}_a^{(e)}(\vartheta) = \overline{u}^{(k)}(\vartheta)$.

3.6. End of cycle 2 with integer variable $k \in \overline{1,K_\vartheta}$.

3.7. The final set $\widetilde{\boldsymbol{U}}^{(e)}(w_a^{(e)}(\vartheta)) = \{\widetilde{u}_a^{(e)}(\vartheta)\}$ and number $\widetilde{\mathbf{\Phi}}^{(e)}_{\overline{\vartheta,T}} = \boldsymbol{F}^{(e)}(T)$ are remembered.

Step 4. Formation of the strategy of optimal adaptive control.

4.1. The selection of any control $\{\widetilde{u}_a^{(e)}(\vartheta)\} \in \widetilde{\boldsymbol{U}}^{(e)}(w_a^{(e)}(\vartheta))$ of the manager P corresponding to the period of time ϑ is carried out.

4.2. The $(\vartheta+1)$-position $w_a^{(e)}(\vartheta+1) = \{\vartheta+1, \overline{x}_a^{(e)}(\vartheta+1)\} \in \boldsymbol{W}(\vartheta, w_a^{(e)}(\vartheta), \vartheta+1, \widetilde{u}_a^{(e)}(\vartheta)) \subseteq \hat{\boldsymbol{W}}(\vartheta+1)$ of the manager P is calculated, where $\overline{x}_a^{(e)}(\vartheta+1) = \varphi_{\overline{\vartheta,\vartheta+1}}(\vartheta+1; \overline{x}_a^{(e)}(\vartheta), \widetilde{u}_a^{(e)}(\vartheta) \in \overline{\boldsymbol{X}}_1(\vartheta+1)$.

4.3. Relied upon $w_a^{(e)}(\vartheta) = \{\vartheta, \overline{x}_a^{(e)}(\vartheta)\} = w_a^{(e)}(\vartheta+1)$.

4.4. End of cycle 1 by integer variable $\vartheta \in \overline{0,T-1}$.

Step 5. Formation of the results of the implementation of the optimal adaptive control strategy.

5.1. A set $\widetilde{u}_a^{(e)}(\cdot) = \{\widetilde{u}_a^{(e)}(\vartheta)\}_{\vartheta\in\overline{0,T-1}}$ is formed which is an implementation on the time interval $\overline{0,T}$ of the control of manager P, generated by the implementation of his strategy of optimal adaptive control $\widetilde{\boldsymbol{U}}_a^{(e)} \in \overline{\boldsymbol{U}}_a^*$, since $\forall\vartheta \in \overline{0,T-1} : \widetilde{u}_a^{(e)}(\vartheta) \in \widetilde{\boldsymbol{U}}_a^{(e)}(w_a^{(e)}(\vartheta))$.

5.2. A set $\overline{x}_a^{(e)}(\cdot) = \{\overline{x}_a^{(e)}(\vartheta)\}_{\vartheta\in\overline{0,T}} = \varphi_{\overline{0,T}}(\cdot; \overline{x}_0, \widetilde{u}_a^{(e)}(\cdot))$ is formed as realization on the time interval $\overline{0,T}$ of the phase trajectory of the considered dynamic system (1)–(4), corresponding to the pair $(\overline{x}_0, \widetilde{u}_a^{(e)}(\cdot))$ and generated by the implementation on the time interval $\overline{0,T}$ of the strategy of optimal adaptive control $\widetilde{\boldsymbol{U}}_a^{(e)} = \overline{\boldsymbol{U}}_a^{(e)} \in \overline{\boldsymbol{U}}_a^*$ by manager P.

5.3. A real number $\widetilde{\mathbf{\Phi}}^{(e,a)}_{\overline{0,T}} = \widetilde{c}^{(e,a)}_{\mathbf{\Phi}_{\overline{0,T}}}(\overline{0,T}, w_0) = \widetilde{\mathbf{\Phi}}^{(e,a)}_{\overline{T-1,T}} = c^{(e,a)}_{\mathbf{\Phi}_{\overline{0,T}}}(\overline{0,T}, w_0) = \mathbf{\Phi}^{(e,a)}_{\overline{0,T}}$ is formed — the optimal value of the result of the process of adaptive terminal control of the manager P on the time interval $\overline{0,T}$ for the discrete dynamic system (1)–(4), (6) relative to its initial position w_0 and functional $\mathbf{\Phi}_{\overline{0,T}}$, i.e. a number satisfying relation (8).

5.4. The obtained results are displayed in a form convenient for the manager P.

The end of the algorithm.

Note that the elements obtained using the described algorithm, $\widetilde{u}_a^{(e)}(\cdot) = \{\widetilde{u}_a^{(e)}(\vartheta)\}_{\vartheta\in\overline{0,T-1}}$, $\overline{x}_a^{(e)}(\cdot) = \{\overline{x}_a^{(e)}(\vartheta)\}_{\vartheta\in\overline{0,T}} = \varphi_{\overline{0,T}}(\cdot; \overline{x}_0, \widetilde{u}_a^{(e)}(\cdot))$ and $\widetilde{\mathbf{\Phi}}^{(e,a)}_{\overline{0,T}} = \widetilde{c}^{(e,a)}_{\mathbf{\Phi}_{\overline{0,T}}}(\overline{0,T}, w_0)$ satisfies relations (15) and, by virtue of the validity of Statement, are the solution of the considered Problem 2 — the optimal adaptive control of the number of employees and the sales system of the Bank's Retail Unit in the presence of a vector objective function of the form (6).

RESULTS OF COMPUTER SIMULATION OF THE SOLUTION OF THE OPTIMAL ADAPTIVE CONTROL PROBLEM

The proposed algorithm for solving the problem of optimal adaptive control of the number of employees of the Retail Bank Unit and their sales system was the basis for the development of an appropriate computer simulation system. This system was developed in the Delphi 7 software environment and allows the formation of the optimal solution for the problem under consideration with various input data defining the control resources.

Consider the results of computer simulation of the problem on a specific practical example, which is a special case of the general economic and mathematical model (1)–(4) with a vector quality criterion (6).

Let the period of application of the control action is $\tau = 3$ (months) and the integer implementation interval of the management process is $\overline{0, 18}$.

The initial values of the following parameters are specified:

$H = (10.75, 5.0, 13.0, 6.2, 5.0, 7.0, 8.2, 3.0)$;

$S = (150, 1827, 720, 38, 280, 68, 340, 50)$;

$u(0) = (0, 0, 0, 0, 0, 0, 0)$;

$x(0) = (140000000, 160000000, 35000000, 42000000, 300000000, 70000000, 140000000, 80000000)$;

$y(0) = (1636, 215, 250, 155, 42, 35, 1830)$.

The set of the indices of the vector coordinates $x(t)$, which determine the parameters of the bank's liabilities portfolio, consist from following numbers: $B = \{5, 6, 7, 8\}$.

At the time $t = 0$ the matrix of sales standards $A(0) = A^{(k)}(0) \in A_1(\tau_t), k \in \overline{1, 3}$ has the following concrete form:

$$\begin{pmatrix} 6 & 8 & 6 & 23 & 23 & 4 & 28 & 0 \\ 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 12 & 0 & 23 & 23 & 4 & 0 & 0 \\ 6 & 12 & 0 & 23 & 23 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 315 \\ 0 & 19 & 6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 23 & 23 & 27 & 28 & 0 \end{pmatrix}$$

FIGURE 1. The matrix $A(0)$

The form of each of matrices $A^{(k)}(\tau_t)$, $k \in \overline{1, 3}$, is shown in Figures 2–4.

$$\begin{pmatrix} 19 & 3 & 6 & 23 & 23 & 4 & 28 & 0 \\ 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 \\ 30 & 3 & 0 & 23 & 23 & 4 & 0 & 0 \\ 30 & 3 & 0 & 23 & 23 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 315 \\ 0 & 3 & 47 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 23 & 23 & 27 & 28 & 0 \end{pmatrix}$$

FIGURE 2. The matrix $A^{(1)}(\tau_t)$

$$\begin{pmatrix} 6 & 8 & 6 & 23 & 23 & 4 & 28 & 0 \\ 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 12 & 0 & 23 & 23 & 4 & 0 & 0 \\ 6 & 12 & 0 & 23 & 23 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 315 \\ 0 & 19 & 6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 23 & 23 & 27 & 28 & 0 \end{pmatrix}$$

FIGURE 3. The matrix $A^{(2)}(\tau_t)$

$$\begin{pmatrix} 6 & 3 & 6 & 23 & 23 & 12 & 28 & 0 \\ 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 3 & 0 & 23 & 23 & 21 & 0 & 0 \\ 6 & 3 & 0 & 23 & 23 & 21 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 315 \\ 0 & 3 & 47 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 23 & 23 & 27 & 28 & 0 \end{pmatrix}$$

FIGURE 4. The matrix $A^{(3)}(\tau_t)$)

The results of computer simulation of the problem of optimal program control of the number of employees of the Retail Bank Unit and their sales system for the period of time $\overline{0, 18}$ are presented in Tables 1–2.

TABLE 1. The optimal implementation of the management of the input of the number of employees

$u^{(e)}(\tau_t)$	τ_t		
	0	1	2
$u_1^{(e)}(\tau_t)$	0	163	179
$u_2^{(e)}(\tau_t)$	0	21	19
$u_3^{(e)}(\tau_t)$	0	25	27
$u_4^{(e)}(\tau_t)$	0	15	17
$u_5^{(e)}(\tau_t)$	0	4	4
$u_6^{(e)}(\tau_t)$	0	3	3
$u_7^{(e)}(\tau_t)$	0	183	201

TABLE 2. The optimal implementation of employee sales management standards

τ_t	0	1	2
$A^{(e)}(\tau_t)$	$A^{(2)}(\tau_t)$	$A^{(1)}(\tau_t)$	$A^{(1)}(\tau_t)$

For these implementations of controls, the value of the vector functional (6) is maximum and equal $\mathbf{\Phi}^{(e)}_{\overline{0,18}} =$ 0.5548.

Let in the time period $\vartheta = 1$ the vector components $x(\vartheta)$ take the following values:

$x(1) = (60000000, 343074227, 46626870, 46008009, 352606724, 60000000, 245354167, 70320728)$.

Then on the interval $\overline{6,18}$ the control actions presented in Tables 3 and 4 will be optimal. For these implementations of controls, the value of the vector functional (6) is equal $\mathbf{\Phi}^{(e,a)}_{\overline{6,18}} = 0.3147$, while the value of this functional without adaptation (with the previous program control) is equal to $\mathbf{\Phi}_{\overline{6,18}} = 0.2538$.

For the considered dynamic system(1)-(4), (6) the optimal trajectory corresponds to the optimal adaptive control, for which the value of the quality criterion for the realization of the process in question at the final time is maximum compared to similar values for other allowable trajectories and program controls.

TABLE 3. The optimal implementation of the management of the input of the number of employees

$u^{(e)}(\tau_t)$	τ_t	
	1	2
$u_1^{(e)}(\tau_t)$	163	179
$u_2^{(e)}(\tau_t)$	-21	-19
$u_3^{(e)}(\tau_t)$	25	27
$u_4^{(e)}(\tau_t)$	15	-17
$u_5^{(e)}(\tau_t)$	4	4
$u_6^{(e)}(\tau_t)$	3	3
$u_7^{(e)}(\tau_t)$	183	201

TABLE 4. The optimal implementation of employee sales management standards

τ_t	1	2
$A^{(e)}(\tau_t)$	$A^{(3)}(\tau_t)$	$A^{(3)}(\tau_t)$

CONCLUSIONS

For the organization of the optimal adaptive terminal control of the number of employees and the sales system of the Bank's Retail Unit in the presence of a vector objective function, i.e. solving the Problem 2 in the chosen class of admissible adaptive control strategies, a recurrent algorithm is proposed that reduces the original multi-step problem to the realization of a finite sequence of auxiliary Problems 1 — optimal program control. In turn, the solution of each of Problem 1 is reduced to the realization of a finite sequence of only one-step algebraic operations and the finding of solutions to finite discrete optimization problems. Then it can be argued that the solution of the problem in question 2 was reduced to the realization of a finite sequence of only one-step algebraic operations and the finding of solutions to finite discrete optimization problems.

The proposed general scheme for solving Problem 2 and the corresponding numerical algorithm made it possible to develop a software package for implementing computer simulation of the solution of formulated Problem 2 — optimal adaptive control of the number of employees and the sales system of the Bank's Retail Unit in the presence of a vector objective function described by an economic-mathematical model (1)–(4), (6).

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